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Spin splitting of the Landau levels and exchange interaction of a non-ideal two-dimensional electron gas in $In_xGa_{1-x}As/InP$ heterostructures

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Abstract. Shubnikov–de Haas oscillations corresponding to the spin-resolved Landau levels of the two-dimensional electron gas at an In_{0.53}Ga_{0.47}As–InP heterointerface were studied in wide ranges of magnetic fields (up to 22 T) and tilt angles. Dependences of the spin splitting Δ_s on the parallel component of magnetic field were investigated for half-filled Landau levels $0\uparrow$, $1\downarrow$, $1\uparrow$, $2\downarrow$ and $2\uparrow$. The exchange interaction was shown to be strongly dependent on the broadening and overlapping of adjacent Landau levels. On the basis of this model, the experimental magnetic field dependences of Δ_s were described. As a result, the absolute value of bare *g*-factor $g_0 = 2.9$, exchange energy $E_x = 30$ meV and Landau level broadening $\Gamma = 7-11$ meV were obtained.

1. Introduction

The spin splitting of Landau levels in a two-dimensional electron gas (2DEG) has been investigated experimentally in $In_x Ga_{1-x} As/In_y Al_{1-y} As$ [1, 2] and $Al_z Ga_{1-z} As/GaAs$ heterostructures [3, 4]. The Zeeman spin splitting has been shown to be modified by the exchange interaction which depends on the difference between the occupancies of the spinup and spin-down levels. It results in the observed enhancement and oscillatory behaviour of the effective g-factor. However, after the cited papers but up to now, almost no new results on the exchange interaction in 2DEG have been published. This may result from certain limitations of the experimental methods and data processing. The effective g*-factor was determined in [1–4] as a ratio of the deduced spin-splitting energies and the total magnetic field B_{tot} , and the g* versus B_{tot} dependence was analysed. Such an approach implies that the exchange interaction and the Zeeman terms depend on the magnetic field in the same way and, hence, g* depends on B_{tot} only, rather than on the parallel magnetic field component B_p . We think that this point needs clarification or even modification.

To determine the Landau level spin splitting, the coincidence method and the magnetotransport activation method are usually used. When applying the coincidence method, a significant overlap of the spin-split levels with different Landau numbers and different spin polarizations occurs. This should have a profound influence on the difference between the occupancies of the spin-up and spin-down levels and on the exchange interaction

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Figure 1. A fragment of the SdH resistivity oscillation versus normal component of magnetic field at four tilt angles θ . Two maxima correspond to the spin–split Landau levels $n_L = 1\uparrow$ and $1\downarrow$. Arrows demonstrate the positions of the oscillation maxima for these two Landau levels at different θ .

magnitude. Application of the activation method also causes another problem. Usually when the overlap of the spin-split levels is significant, the conductivity minima of the Shubnikov– de Haas (SdH) oscillations are determined by hopping processes rather than by activation to the delocalized states [5].

In this paper a new method is suggested to determine the Landau level spin splitting and its dependence on the normal and parallel magnetic field components. The method allows us to investigate details of the exchange interaction causing the effective g-factor enhancement in high magnetic fields for a 2DEG in $In_{0.53}Ga_{0.47}As/InP$ heterostructures.

2. Samples and experimental methods

Selectively doped In_{0.53}Ga_{0.47}As/InP 2DEG heterostructures grown by liquid-phase epitaxy [6] on semi-insulating InP(100) substrates were used in the experiment. The samples consisted of an InP buffer layer of thickness $d = 1 \ \mu$ m, an n-doped InP layer ($n = 2 \times 10^{17} \text{ cm}^{-3}$, $d = 0.3 \ \mu$ m) and an undoped covering In_{0.53}Ga_{0.47}As layer ($d = 0.3 \ \mu$ m).



Figure 2. Effective g-factor versus reciprocal cosine of θ for different spin-resolved Landau levels. Full and open symbols correspond to the spin-up and spin-down Landau levels, respectively.

The undoped layers were obtained by adding small amounts of Sm to the melt during epitaxial growth. DC magnetoresistance measurements of the 2DEG parameters were performed on standard Hall bar samples. Ohmic contacts to the structures were formed by alloying indium in vacuum at 400 °C. The sample holder allowed us to change the tilt angle θ between the magnetic field and the sample normal. The magnetic field dependences of the diagonal resistivity R_{xx} and Hall resistivity R_{xy} were measured for different tilt angles at 4.2 K in magnetic fields up to 22 T. The exact values of θ were determined from the magnetic field dependence of the R_{xy} at low magnetic field where R_{xy} is proportional to the normal field component $B_n = B_{tot} \cos \theta$.

3. Experimental results

Typical B_n dependences of the diagonal resistivity at different values of θ are presented in figure 1 for one of the samples (sample 258) characterized by a 2DEG density $n_s = 3.7 \times 10^{11}$ cm⁻² and mobility $\mu = 3.5 \times 10^4$ cm² V⁻¹ s⁻¹ at 4.2 K. One can see that, for different values of θ , maxima of the SdH oscillations corresponding to spinsplit Landau levels are placed at different B_n . This is the basis for the proposed method for obtaining the spin-splitting energy Δ_s . First let us suppose that $\Delta_s \ll \hbar \omega_c$. Then the normal magnetic field component B_{n0} corresponding to the maxima of SdH oscillations can be obtained from the following equation:

$$E_F = (n_L + \phi_0)\hbar\omega_c(B_{n0}) \tag{1}$$

where E_F is the Fermi energy, $\hbar\omega_c(B_{n0}) = e\hbar B_{n0}/m^*$ is the cyclotron energy, n_L is the Landau level number and $\phi_0 \simeq 0.5$ is the phase constant. If $\Delta_s \neq 0$, equation (1) can be rewritten as

$$E_F = (n_L + \phi_0)\hbar\omega_c(B_{n1}) \pm \frac{1}{2}\Delta_s(B_{n1}, B_{p1}).$$
(2)

We have taken into account the fact that the spin-splitting energy might depend on both normal and parallel magnetic field components. Equation (2) is usually written in the following form:

$$E_F = (n_L + \phi_0)\hbar\omega_c(B_{n1}) \pm \frac{1}{2}g^*\mu_B B_{tot1}$$
(3)

or

$$\frac{E_F}{\hbar\omega_c} = (n_L + \phi_0) \pm \frac{1}{4} g^* \frac{m^*}{m_0} \frac{1}{\cos\theta}.$$
(3a)

Therefore, if the effective g^* -factor is a constant independent of the tilt angle θ , the $1/B_n$ corresponding to the maxima of SdH oscillations would depend linearly on the reciprocal cosine of θ . Treatment of the experimental angular dependence of $1/B_n$ corresponding to different n_L shows that equation (3*a*) does not fit the experimental data well. So, we have to suppose that g^* is a function of the tilt angle and find it as

$$g^* = \left| \frac{E_F}{\hbar \omega_c} - (n_L + \phi_0) \right| \frac{4m_o}{m^*} \cos \theta.$$
(4)

The results are presented in figure 2. The effective g^* -factor for the $0\uparrow$ sublevel is much greater than those for other Landau levels and decreases strongly with increasing tilt angle. For the $2\uparrow$ and $2\downarrow$ sublevels the g^* -factor increases with increasing θ , while at $n_L = 1$ it exhibits a non-monotonic dependence on the tilt angle. This complicated picture seems to be due not only to physical reasons but also to the method of analysing the experimental data. In our opinion, it is more appropriate to analyse the spin splitting in terms of splitting energy depending, generally, on both normal and parallel magnetic field components:

$$\Delta_s(B_{n1}, B_{p1}) = \pm 2(n_L + \phi_0)(B_{n1} - B_{n0})e\hbar/m^*$$
(5)

rather than in terms of an effective g-factor. Here B_{n1} and B_{n0} are the experimental values of the normal magnetic field corresponding to the positions of the SdH maximum with and without spin splitting, respectively. To obtain B_{n0} we examine the SdH oscillations at low magnetic fields (large Landau numbers) were the spin splitting is not resolved. The experimentally obtained dependence of the Landau numbers on the reciprocal normal magnetic field corresponding to non-spin-split SdH oscillation maxima is shown in figure 3. The straight line in figure 3 is the best fit calculated from equation (1). It gives $\phi_0 = 0.54$, close to the theoretical value. The values of B_{n0} corresponding to small Landau numbers



Figure 3. Dependence of the Landau numbers on the reciprocal value of normal magnetic field corresponding to non-spin-split SdH oscillation maxima.

can be obtained from this line (see figure 3). Using these values and equation (5), we have determined Δ_s for some Landau levels. The dependences of these values on B_p are shown in figure 4.

It can be seen that, for $n_L = 0$ and high B_n , Δ_s is larger than for higher n_L and smaller B_n but rises with the parallel magnetic field slowly. For $n_L = 1$ and the intermediate value of B_n , the value of Δ_s varies more rapidly at low and high B_p than at intermediate B_p . For $n_L = 2$ (small B_n), the value of Δ_s rises quickly with increasing B_p . So, the spin splitting depends on the parallel and normal magnetic field components in a different and non-trivial way.

4. Analysis of experimental results

To analyse our experimental data, it is necessary to take into account previous results [3,4] demonstrating the modification of the spin splitting of Landau levels in 2DEG by exchange interactions of electrons given by [7]

$$\Delta_s = g_0 \mu_B B_{tot} + E_x \left| \frac{(n_{\uparrow} - n_{\downarrow})}{n_s} \right| \tag{6}$$

where g_0 is the bare g-factor, E_x is the exchange energy and $n_{\uparrow} - n_{\downarrow}$ is the difference between the occupancies for the two spin states. It is clear from equation (6) that Δ_s has its maximum when the Fermi level lies between the spin-up and the spin-down sublevels of the same Landau level, i.e. $n_{\uparrow} - n_{\downarrow} \simeq v_L$ ($v_L = eB_n/2\pi\hbar$ is the Landau level degeneracy), and has its minimum when the Fermi level lies between different Landau levels, and $n_{\uparrow} - n_{\downarrow} = 0$. However, we are interested in the intermediate case when the Fermi level lies in one of the spin-split sublevels (maximum of the SdH oscillations). In this case $n_{\uparrow} - n_{\downarrow} = 0.5v_L$ for



Figure 4. Dependence of the spin-splitting value on the parallel component of magnetic field for five spin-resolved Landau levels.

an ideal 2DEG but for a non-ideal 2DEG (see figure 5) it can be written in the following form:

$$|n_{\uparrow} - n_{\downarrow}| = (0.5 - \delta_0 + \delta_1)\nu_L \tag{7}$$

where δ_0 and δ_1 are corrections caused by the overlapping of the neighbouring spin-split sublevels with the same and different Landau indices, respectively (figure 5). These correction terms depend strongly on the thermal and collision broadening of the Landau levels and decrease with increase in Landau level energy separation. In our case the thermal broadening $k_B T = 0.36$ meV is much less than the collision broadening $\Gamma = 7.2 \pm 0.6$ meV of Landau levels estimated from the amplitude of the SdH oscillations amplitude in low magnetic fields and can be ignored. The overlapping parameters δ_0 and δ_1 depend on the shape of Landau level broadening. As a first approximation, we assume the Gaussian density of states for broadened Landau levels which gives

$$\delta_0 = \frac{1}{2} \left(1 - \operatorname{erf}\left(-\frac{\sqrt{2\Delta_s}}{\Gamma} \right) \right) \tag{8}$$



Figure 5. Density of states for spin-split Landau levels in a non-ideal 2DEG.

$$\delta_1 = \frac{1}{2} \left(1 - \operatorname{erf} \left(-\frac{\sqrt{2}(\hbar\omega_c - \Delta_s)}{\Gamma} \right) \right).$$
(9)

For the Fermi level coinciding with a spin-split Landau level, our equations give

$$\Delta_s = g_0 \mu_B B_{tot} + \frac{E_x e B_n}{n_s h} (\frac{1}{2} - \delta_0 + \delta_1).$$
(10)

To analyse the dependence of Δ_s on magnetic field components, we consider various limiting cases.

(a) $\Delta_s \gg \Gamma/2$, $\hbar\omega_c - \Delta_s \gg \Gamma/2$, i.e. the broadening and overlapping of the Landau levels is small. In this case, realized in a high-quality 2DEG and/or high magnetic field, equation (10) can be rewritten as

$$\Delta_s(B_n, B_p) = g_0 \mu_B B_n \left(1 + \left(\frac{B_p}{B_n} \right)^2 \right)^{0.5} + \frac{E_x e B_n}{2n_s h}.$$
 (11)

In this case the spin splitting is large, even for $B_p = 0$, undergoing further slow increase with increasing B_p .

(b) $\hbar\omega_c \gg \Delta_s \simeq \Gamma/2$, i.e. the overlap of spin-split sublevels only with the same Landau index is important. In this case of intermediate normal magnetic field, equation (10) gives

$$\Delta_s(B_n, B_p) = g_0 \mu_B B_n \left(1 + \left(\frac{B_p}{B_n}\right)^2 \right)^{0.5} + \frac{E_x e B_n}{2n_s h} \operatorname{erf}\left(\sqrt{2} \frac{\Delta_s(B_n, B_p)}{\Gamma}\right).$$
(12)

In this case the value of Δ_s depends on B_p in an implicit form and this dependence is seen to be stronger than in case (a).

(c) $\Delta_s \gg \hbar \omega_c - \Delta_s \simeq \Gamma/2$, i.e. the overlap of spin-split sublevels of the same Landau level is negligible, with noticeable overlap of sublevels belonging to adjacent Landau levels. Here

$$\Delta_s(B_n, B_p) = g_0 \mu_B B_n \left(1 + \left(\frac{B_p}{B_n}\right)^2 \right)^{0.5} + \frac{E_x e B_n}{2n_s h} \left(2 - \operatorname{erf}\left(\sqrt{2} \frac{e\hbar B_n / m^* - \Delta_s(B_n, B_p)}{\Gamma}\right) \right).$$
(13)

(d) $\Delta_s \gg \Gamma/2 \gg \hbar \omega_c - \Delta_s$. This is the case of almost coincidence of the neighbouring Landau levels and

$$\Delta_s(B_n, B_p) = g_0 \mu_B B_n \left(1 + \left(\frac{B_p}{B_n} \right)^2 \right)^{0.5} + \frac{E_x e B_p}{n_s h}.$$
 (14)

The exchange term in this case exceeds that of the pure case (a) (equation (11)) by a factor of 2 and has the same magnetic field dependence. This means that it is necessary to take into account the overlapping of the Landau levels to obtain the correct value of the exchange energy from the coincidence method.

Let us compare the experimental values of Δ_s with the calculations.

(1) $n_L = 0\uparrow$: $\hbar\omega_c \simeq 28.8 \text{ meV}$, $\Delta_s \simeq 11 \text{ meV}$ (figure 6(*a*)). The value of Γ can be estimated from the amplitude of sinusoidal SdH oscillations at low magnetic fields as $\Gamma = \hbar/\tau_q = 7.2 \text{ meV}$ (τ_q is the one-particle relaxation time). Therefore, in this case, equation (11) can be used to describe the dependence of Δ_s on B_p with B_n assumed to be constant because its change is much less than that of B_p . The values of E_x and g_0 are used as fitting parameters and found to be $E_x = 30 \pm 1 \text{ meV}$ and $g_0 = 2.9 \pm 0.5$.

(2) $n_L = 1 \downarrow$: $\hbar \omega_c \simeq 15.5 \text{ meV}$, $\Delta_s \simeq 3-7 \text{ meV}$ (figure 6(b)). The dotted curve in figure 6(b) is obtained from equation (11) with the parameters obtained above for $n_L = 0$. The fitting is reasonable only at high magnetic fields. This means that, for small values of the parallel magnetic field, Δ_s is comparable with the broadening of the Landau levels and cannot be ignored. So we must use equation (12) rather than equation (11) with Γ as a fitting parameter. The solid curve in figure 6(b) represents the corresponding $\Delta_s(B_p)$ dependence with $\Gamma = 10$ meV and is seen to give a good fit all the experimental data.

(3) $n_L = 1\uparrow$: $\hbar\omega_c \simeq 11.9 \text{ meV}$, $\Delta_s \simeq 4-8 \text{ meV}$ (figure 6(c)). The dotted and solid curves in figure 6(c) were obtained in the same way as for the 1 \downarrow level, using the same fitting parameters ($E_x = 30 \text{ meV}$, $g_0 = 2.9 \text{ and } \Gamma = 11 \text{ meV}$). Most of the data can be easily seen to be described by equation (12) taking into account the population correction terms connected with the overlapping of 1 \downarrow and 1 \uparrow levels. However, a noticeable discrepancy between the calculated and experimental data exists at high B_p where $\hbar\omega_c - \Delta_s \leq \Delta_s \simeq 6 \text{ meV}$ is comparable with $\Gamma/2$. This means that it is necessary to take into account the overlapping of $2\downarrow$ and $1\uparrow$ sublevels as well and to use the complete equation (10) to evaluate the value of Δ_s . The results of this calculation are presented in figure 6(c) by the broken curve. Good fitting can be obtained only if we take a slightly different value of Γ for $2\downarrow$ sublevel: $\Gamma_2 = 7 \text{ meV}$.

(4) $n_L = 2$ spin up and down: $\hbar \omega_c \simeq 6-9$ meV, $\Delta_s \simeq 2-6$ meV (figure 6(d)). The large error in the value of Δ_s for this Landau level (see figure 6(d)) is connected with the high value of the tilt angle close to 90°. As a result, a small error in the tilt angle leads to a large error in B_{n1} and Δ_s . On the other hand, the large value of θ leads to a large spin splitting comparable with $\hbar \omega_c$ and to a strong overlapping between neighbouring



Figure 6. Comparison of the experimental and calculated dependences of spin splitting on the parallel component of magnetic field for (*a*) the $0\uparrow$ Landau level, (*b*) the $1\downarrow$ Landau level, (*c*) the $1\uparrow$ Landau level and (*d*) the $2\uparrow$ and $2\downarrow$ Landau levels.

levels with different Landau numbers. The calculated curves in figure 6(d) were obtained in the same way as for the $1\downarrow$ sublevel (compare figures 6(c) and 6(d)). The comparison of



Figure 6. (Continued)

experimental and theoretical data in figure 6(d) shows the necessity to take into account the overlapping of both of spin sublevels of the same Landau number (it plays the main role

at B_p ; see the solid curve in figure 6(d)) and of adjacent Landau levels (it plays the main role at high B_p ; see the broken curve in figure 6(d)). To get the best fit, we must assume different broadening parameters for levels with the same Landau index $\Gamma_1 = 10$ meV and with different Landau index $\Gamma_2 = 7$ meV.

Therefore, all features of the magnetic field dependences of spin splitting can be described from a single point of view: taking into account the broadening and overlapping of spin-split Landau levels resulted from 2DEG disorder. As a result, we have obtained characteristic parameters of the spin-splitting of the Landau levels in the 2DEG at the In_{0.53}Ga_{0.47}As–InP heterointerface. The absolute value of the bare *g*-factor not modified by many-particle effects is $g_0 = 2.9$, the exchange energy $E_x = 30$ meV and the Landau level broadening $\Gamma = 7-11$ meV.

The value obtained for the bare g-factor is smaller than the value $g_0 = 3.38$ calculated in [8]. However, the latter is to be reduced by non-parabolicity [9]. Our result is close to the value $g_0 = 3$ determined earlier from galvano/magnetic measurements [1], but smaller than g-factor for bulk In_{0.53}Ga_{0.47}As, $g_0 = 4.1$, obtained in [10] by electrically detected spin resonance in a 2DEG. Recent investigations of the optically detected magnetic resonance of the undoped InP/In_xGa_{1-x}As quantum wells [11] have given for the quasi-three-dimensional case the isotropic value $g_0 = 4.01$ and demonstrated the confinement anisotropy of g_0 , reducing it below $g_0 \approx 2$ for a well width of less than 10 nm.

Our value of Landau level broadening, $\Gamma = 7-10$ meV, is in good agreement with the value $\Gamma = \hbar/\tau_q = 7.2$ meV obtained from the amplitude of the low-field SdH oscillation, contrary to the results of [3]. The densities of states for half-filled and almost filled (or empty) Landau levels have been found to differ, in agreement with the theoretical prediction for a disordered conductor [12].

We have not found any literature data for the exchange energy E_x in a 2DEG at the In_{0.53}Ga_{0.47}As–InP heterointerface. The only published value, $E_x = 2.8$ meV, was measured in Al_zGa_{1-z}As/GaAs structures [3, 4]. Our results show that many-particle effects are of more significance for spin-dependent effects in a 2DEG at the InP–In_{0.53}Ga_{0.47}As heterointerface rather than in Al_zGa_{1-z}As/GaAs heterostructures. This is in agreement with the investigations of weak localization in a 2DEG in the presence of a spin–orbit interaction [13].

5. Conclusion

To summarize, SdH oscillations corresponding to the spin-resolved Landau levels of a 2DEG at an In_{0.53}Ga_{0.47}As–InP heterointerface were studied in wide ranges of magnetic fields (up to 22 T) and tilt angles. The values of spin splitting Δ_s were determined for half-filled Landau levels with the Landau indices $0\uparrow$ (spin up), $1\downarrow$ (spin down), $1\uparrow$, $2\downarrow$ and $2\uparrow$ in a wide range of magnetic field components parallel to heterointerface. The concept of the effective g-factor, $g^* = \Delta_s/\mu_b B_{tot}$, was shown to be inadequate for the spin-splitting characterization since the value of Δ_s depends on the normal and parallel magnetic field components in a different and not trivial way. To analyse the experimental dependences, a modification of the Zeeman spin splitting by the exchange interaction of electrons with different spins was taken into account, in agreement with earlier results. It has been shown for the first time that the broadening and overlapping of Landau levels must be included in the spin splitting treatment in real heterostructures. The overlapping of the levels with the same Landau number and different spins plays the main role at small spin splittings. For the case of large spin splittings it is necessary to take into account the overlapping of levels with different Landau indices. The widely used coincidence method was demonstrated

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not to give adequate results for the *g*-factor and exchange interaction until Landau level overlapping is taken into account.

On the basis of this model the experimental dependences of spin splitting on magnetic field were described. As a result, the characteristic parameters of spin splitting such as the bare *g*-factor, the Landau level broadening and the exchange energy were obtained for a 2DEG in an $In_{0.53}Ga_{0.47}As/InP$ heterostructure.

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